WS 8 — Failing Limits; Properties of Limits

1) a) $\lim_{x\to c}[f(x)\,g(x)] =$

$\left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$

$(5)(-2) = -10$

b) $\lim_{x\to c}[f(x) - g(x)]$

$\lim_{x\to c} f(x) - \lim_{x\to c} g(x)$

$5 - 2 = \boxed{3}$

c) $\lim_{x\to c}\sqrt{h(x)}$

$\sqrt{\lim_{x\to c} h(x)}$

$\sqrt{9} = \boxed{3}$

d) $\lim_{x\to c}\left[\frac{g(x)+1}{x}\right]$

$\frac{\lim_{x\to c} g(x) + \lim_{x\to c} 1}{\lim_{x\to c} x} = \frac{-2+1}{c}$

$= \boxed{-\frac{1}{c}}$

e) $\lim_{x\to c}[2h(x) - 3g(x)]$

$2\lim_{x\to c} h(x) - 3\lim_{x\to c} g(x)$

$2(9) - 3(5) = \boxed{3}$

f) $\lim_{x\to c}\left[\frac{f(x)}{h(x)}\right] = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} h(x)} = \boxed{\frac{5}{9}}$

g) $\lim_{x\to c}\frac{g(x)}{f(x)} = \frac{\lim_{x\to c} g(x)}{\lim_{x\to c} f(x)}$

$= -\frac{2}{5}$

h) $\lim_{x\to c}[g(x)]^2 = \left[\lim_{x\to c} g(x)\right]^2$

$= (-2)^2 = 4$

2) a) $\lim_{x\to 2}[f(x) + g(x)]$

$\lim_{x\to 2} f(x) + \lim_{x\to 2} g(x)$

$1 + 2 = \boxed{3}$

b) $\lim_{x\to 3} f(g(x))$

$f\left[\lim_{x\to 3} g(x)\right]$

$f(0) = \boxed{2}$

c) $\lim_{x\to 3}[f(x)\,g(x)]$

$\lim_{x\to 3} f(x) \cdot \lim_{x\to 3} g(x)$

$1 \cdot 2 = \boxed{2}$

d) $\lim_{x\to 2}[2f(x) + 5g(x)]$

$2\lim_{x\to 2} f(x) + 5\lim_{x\to 2} g(x)$

$2(-1) + 5(2) = \boxed{8}$

e) $\lim_{x\to -1}\left[\frac{f(x)}{g(x)}\right]$

$\frac{\lim_{x\to -1} f(x)}{\lim_{x\to -1} g(x)} = \boxed{\frac{1}{2}}$

f) $\lim_{x\to 2}[x\,f(x)]$

$\lim_{x\to 2} x \cdot \lim_{x\to 2} f(x)$

$2(-1) = \boxed{-2}$

3) a) $\lim_{x \to 7} (g(x) + 5)$

$\lim_{x \to 7} g(x) + \lim_{x \to 7} 5$

$\boxed{10}$

b) $\lim_{x \to 7} x \, f(x)$

$\lim_{x \to 7} x \cdot \lim_{x \to 7} f(x)$

$(7)(0) = \boxed{0}$

c) $\lim_{x \to 7} g^2(x)$

$\left[\lim_{x \to 7} g(x)\right]^2 = 5^2$

$= \boxed{25}$

d) $\lim_{x \to 7} \left[\frac{g(x)}{f(x) - 1}\right]$

$= \frac{\lim_{x \to 7} g(x)}{\lim_{x \to 7} f(x) - \lim_{x \to 7} 1}$

$= \frac{5}{0 - 1} = \boxed{-5}$